Summer Math Review

For

Rising 8th Graders

Eighth grade math classes are designed to move students into the study of algebra. The integration of new and prior knowledge to solve algebraic problems is the major focus, but mathematical thinking and the continuing development of logical reasoning skills are also emphasized. In order for students to be successful, it is critical that students have a firm foundation in skills taught in previous math courses.

This packet is designed to give students additional practice with those skills. The problems in this packet review content areas that are necessary for success in eighth grade math classes. It is important students show their work, including all the steps that led to the solution. This packet is due the first day of school.

Algebra 1
Summer Review Packet

About Algebra 1:

Algebra 1 teaches students to think, reason, and communicate mathematically. Students use variables to determine solutions to real world problems. Skills gained in Algebra 1 are foundational to subsequent math courses.

Summer packet directions:

The problems in this packet are designed to help you review content areas that are important to your success in Algebra 1. **All work is to be shown for each problem.**

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Order of Operations

To avoid having different results for the same problem, mathematicians have agreed on an order of operations when simplifying expressions that contain multiple operations.

1. Perform any operation(s) inside grouping symbols. (Parentheses, brackets, above or below a fraction bar)
2. Simply any term with exponents.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

One easy way to remember the order of operations process is to remember the acronym PEMDAS or the old saying "Please Excuse My Dear Aunt Sally."

P --Perform operations in grouping symbols
E – Simplify exponents
M – Perform multiplication and division in order from left to right
D
A – Perform addition and subtraction in order from left to right
S

Example 1

2 – 3^2 + (6 + 3 x 2)
2 – 3^2 + (6+6)
2 – 3^2 + 12
2 – 9 + 12
-7 + 12
=5

Example 2

-7 + 4 + (2^3 – 8 ÷ -4)
-7 + 4 + (8 – 8 ÷ -4)
-7 + 4 + (8--2)
-7 + 4 + 10
-3 + 10
=7

Order of Operations

Evaluate each expression. Remember your order of operations (PEMDAS).

1. $6 + 4 - 2 \cdot 3 =$

2. $(-2) \cdot 3 + 5 - 7 =$

3. $15 \div 3 \cdot 5 - 4 =$

4. $29 - 3 \cdot 9 + 4 =$

5. $20 - 7 \cdot 4 =$

6. $4 \cdot 9 - 9 + 7 =$

7. $50 - (17 + 8) =$

8. $(12 - 4) \div 8 =$

9. $12 \cdot 5 + 6 \div 6 =$

10. $18 - 4^2 + 7 =$

11. $3(2 + 7) - 9 \cdot 7 =$

12. $3 + 8 \cdot 2^2 - 4 =$

13. $16 \div 2 \cdot 5 \cdot 3 \div 6 =$

14. $12 \div 3 - 6 \cdot 2 - 8 \div 4 =$

15. $10 \cdot (3 - 6^2) + 8 \div 2 =$

16. $6.9 - 3.2 \cdot (10 \div 5) =$

17. $32 \div [16 \div (8 \div 2)] =$

18. $[10 + (2 \cdot 8)] \div 2 =$

19. $180 \div [2 + (12 \div 3)] =$

20. $\frac{1}{4}(3 \cdot 8) + 2 \cdot (-12) =$

21. $\frac{5+[30-(8-1)^2]}{11-2^2} =$

22. $\frac{3[10-(27\div 9)]}{4-7} =$

23. $5(14 - 39 \div 3) + 4 \cdot \frac{1}{4} =$

24. $[8 \cdot 2 - (3 + 9)] + [8 - 2 \cdot 3] =$

25. $162 \div [6(7 - 4)^2] \div 3 =$

Operations with Signed Numbers

Adding and Subtracting Signed Numbers

Adding Signed Numbers

Like Signs		Different Signs	
Add the numbers & carry the sign		Subtract the numbers & carry the sign of the larger number	
$(+)+(+)=+$	$(+\mathbf{3})+(+\mathbf{4})=+\mathbf{7}$	$(+)+(-)=?$	$(+\mathbf{3})+(-\mathbf{2})=+\mathbf{1}$
$(-)+(-)=-$	$(-\mathbf{2})+(-\mathbf{3})=-\mathbf{5}$	$(-)+(+)=?$	$(-\mathbf{5})+(+\mathbf{3})=-\mathbf{2}$

Subtracting Signed Numbers

Don't Subtract! Change the problem to **addition** and change the sign of the **second** number. Then use the addition rules.

$(+9)-(+12)=(+9)+(-12)$	$(+4)-(-3)=(+4)+(+3)$
$(-5)-(+3)=(-5)+(-3)$	$(-1)-(-5)=(-1)+(+5)$

Simplify. *Do not use a calculator for this section.*

1. $9+-4=$
2. $-8+7=$
3. $-14-6=$
4. $-30+-9=$
5. $14-20=$
6. $-2+11=$
7. $20--6=$
8. $7-10=$
9. $-6--7=$
10. $5-9=$
11. $-8-7=$
12. $1--12=$

Multiplying and Dividing Signed Numbers

If the signs are the same, the answer is *positive*.		If the signs are different, the answer is *negative.*	
Like Signs		Different Signs	
$(+)(+) = +$	$(+3)(+4) = +12$	$(+)(-) = -$	$(+2)(-3) = -6$
$(-)(-) = +$	$(-5)(-3) = +15$	$(-)(+) = -$	$(-7)(+1) = -7$
$\frac{+}{+} = +$	$\frac{+12}{+4} = +3$	$\frac{+}{-} = -$	$\frac{+6}{-3} = -2$
$\frac{-}{-} = +$	$\frac{-15}{-3} = +5$	$\frac{-}{+} = -$	$\frac{-7}{+1} = -7$

Simplify. *Do not use a calculator for this section.*

1. $(-5)(-3) =$
2. $\frac{-6}{2} =$
3. $(2)(4) =$
4. $\frac{-12}{-4} =$
5. $(-1)(-5) =$
6. $\frac{-16}{8} =$
7. $\frac{-7}{-1} =$
8. $(3)(-4) =$
9. $\frac{8}{-4} =$
10. $(-2)(7) =$
11. $\frac{-20}{-1} =$
12. $(2)(-5) =$

Rounding Numbers

Step 1: Underline the place value in which you want to round.

Step 2: Look at the number to the right of that place value you want to round.

Step 3: If the number to the right of that place value you want to round is less than 5, keep the number the same and drop all other numbers.

If the number to the right of the place value you want to round is 5 or more, round up and drop the rest of the numbers.

Example: Round the following numbers to the tenths place.

Tenths

1. 23.1246	2 is less than 5 so keep the 1 the same		23.1
2. 64.2685	6 is greater than 5 so add one to the 2		64.3
3. 83.9721	7 is greater than 5 so add one to the 9	$\begin{array}{r} {}^{1} \\ 83.9721 \\ +\ \ 1 \\ \hline 84.0 \end{array}$	84.0

Round the following numbers to the **tenths** place.

1. 18.6231	__________	6. 0.2658	__________
2. 25.0543	__________	7. 100.9158	__________
3. 3.9215	__________	8. 19.9816	__________
4. 36.9913	__________	9. 17.1083	__________
5. 15.9199	__________	10. 0.6701	__________

Evaluating Expressions

Example

Evaluate the following expression when $\boldsymbol{x = 5}$

Rewrite the expression substituting 5 for the x and simplify.

a. $5x =$ $5(5) = 25$
b. $-2x =$ $-2(5) = -10$
c. $x + 25 =$ $5 + 25 = 30$
d. $5x - 15 =$ $5(5) - 15 = 25 - 15 = 10$
e. $3x + 4 =$ $3(5) + 4 = 15 + 4 = 19$

Evaluate each expression given that: x = 5 y =-4 z = 6

1. $3x$
2. $2x^2$
3. $3x^2 + y$
4. $2(x + z) - y$
5. $y + 4$
6. $5z - 6$
7. $xy + z$
8. $2x + 3y - z$
9. $5x - (y + 2z)$
10. $\frac{xy}{2}$
11. $x^2 + y^2 + z^2$
12. $2x(y + z)$
13. $5z + (y - x)$
14. $2x^2 + 3$
15. $4x + 2y - z$
16. $\frac{yz}{2}$

Combining Like Terms

What is a *term*? The parts of an algebraic expression that are separated by an addition or subtraction sign are called *terms*.
The expression $4x + 2y - 3$ has 3 terms.

What are *like terms*? Terms with the same variable factors are called *like terms*.
$2n$ and $3n$ are like terms, but $4x$ and $3y$ are not like terms because their variable factors x and y are different.

To simplify an expression, you must combine like terms.

Examples:
Simplify

1. $5x + 8x$
   $5x + 8x = (5 + 8)x = 13x$

2. $3x + 4 - 2x + 3$
   $3x - 2x + 4 + 3 = (3 - 2)x + 4 + 3 = x + 7$

3. $3y - 6y$
   $3y - 6y = (3 - 6)y = -3y$

4. $2b + 5c + 3b - 6c$
   $2b + 3b + 5c - 6c = (2 + 3)b + (5 - 6)c = 5b - c$

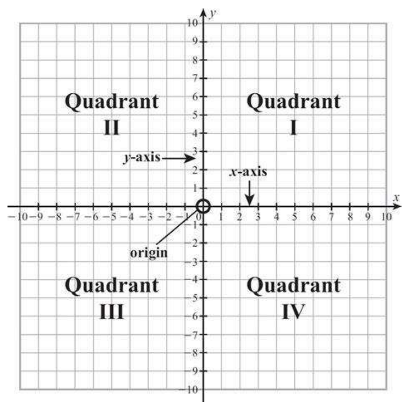
Practice: Simplify each expression

1. $6n + 5n$
2. $25b + 15b$
3. $37z + 4z$
4. $x - 5x$
5. $3n + 1 - 2n + 8$
6. $4f + 5f - 6 + 8$
7. $7t + 9 - 4t + 3$
8. $2k + 4 - 8k - 1$
9. $4r + 3r + 6y - 2y$
10. $8g + 9h - 4g - 5h$
11. $2m + 3n - 4m + 5n$
12. $a + 5b - 2a + 9b$

Graphing

Points in a plane are named using 2 numbers, called a coordinate pair. The first number is called the x-coordinate. The x-coordinate is positive if the point is to the right of the origin and negative if the point is to the left of the origin. The second number is called the y-coordinate. The y-coordinate is positive if the point is above the origin and negative if the point is below the origin.

The x-y plane is divided into 4 quadrants (4 sections) as described below.

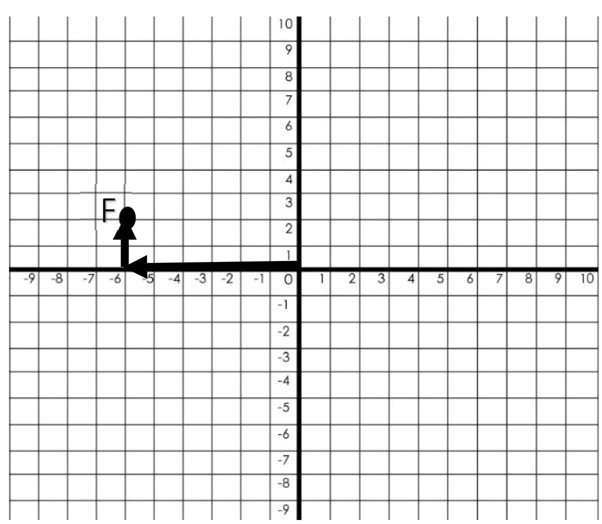


All points in Quadrant 1 have a **positive x**-coordinate and a **positive y**-coordinate (+x, +y).
All points in Quadrant 2 have a **negative x**-coordinate and a **positive y**-coordinate (-x, +y).
All points in Quadrant 3 have a **negative x**-coordinate and a **negative y**-coordinate (-x,-y).
All points in Quadrant 4 have a **positive x**-coordinate and a **negative y**-coordinate (+x,-y).

Plot each point on the graph below. Remember, coordinate pairs are labeled (x,y). Label each point on the graph with the letter given.

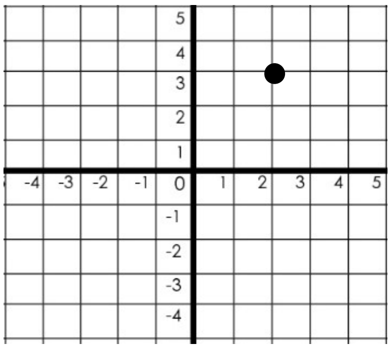
1. A(3,4) 2. B(4,0) 3. C(-4,2) 4. D(-3,-1) 5. E(0,7)

Example: F(-6,2)

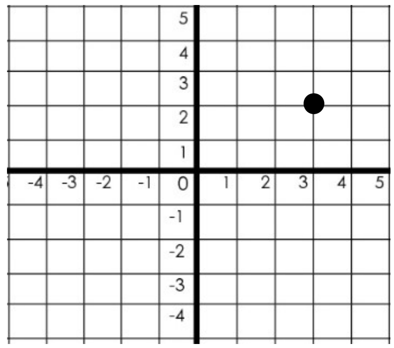


Determine the coordinate for each point below:

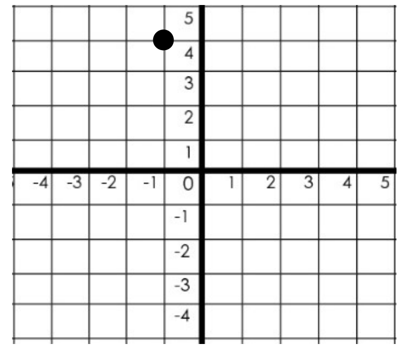
Example (2 , 3)



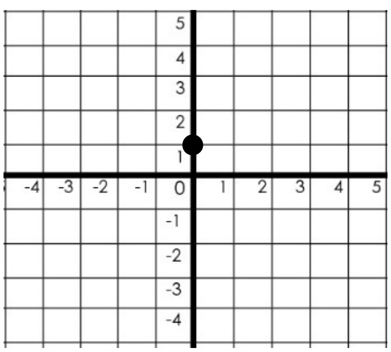
6. (___ , ___)



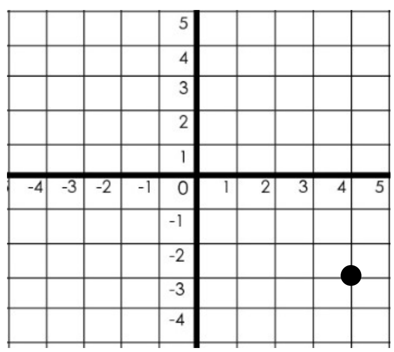
7. (___ , ___)



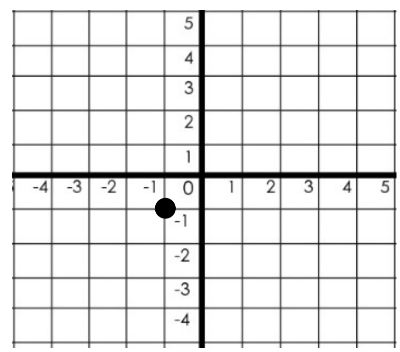
8. (___, ___)



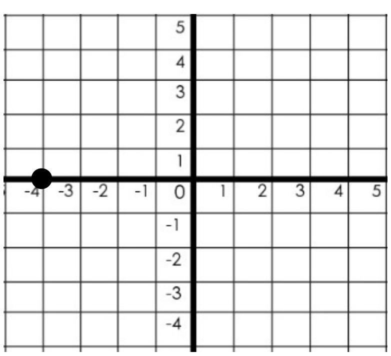
9. (___ , ___)



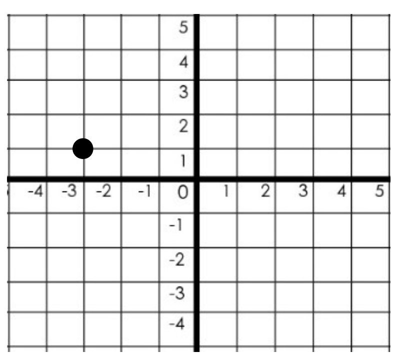
10. (___ , ___)



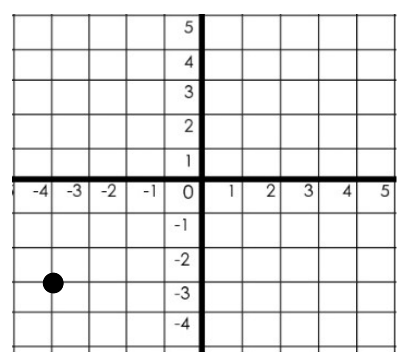
11. (___ , ___)



12. (___ , ___)



13. (___ , ___)



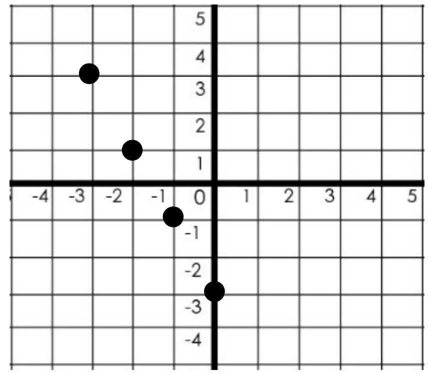
Complete the following tables. Then graph the data on the grid provided.

Example: $\boldsymbol{y = -2x - 3}$

$\boldsymbol{x}$	$\boldsymbol{y}$
-3	3
-2	1
-1	-1
0	-3

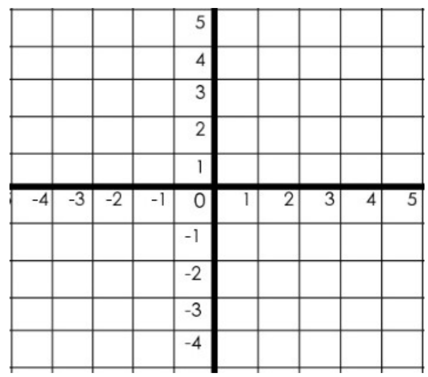
Work:

$$x = -3$$
$$y = -2(-3) - 3 = 6 - 3 = 3$$
Therefore $(\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{-3}, \boldsymbol{3})$
$$x = -2$$
$$y = -2(-2) - 3 = 4 - 3 = 1$$
Therefore $(\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{-2}, \boldsymbol{1})$
$$x = -1$$
$$y = -2(-1) - 3 = 2 - 3 = -1$$
Therefore $(\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{-1}, \boldsymbol{-1})$
$$x = 0$$
$$y = -2(0) - 3 = 0 - 3 = -3$$
Therefore $(\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{0}, \boldsymbol{-3})$



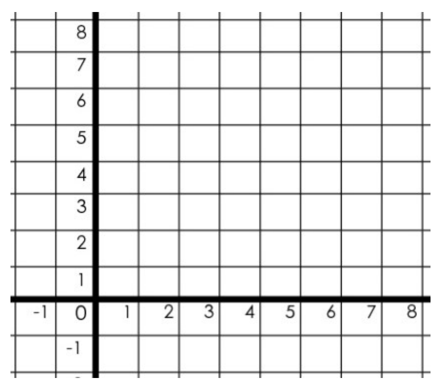
14. $y = x + 2$

$\boldsymbol{x}$	$\boldsymbol{y}$
0	
1	
2	



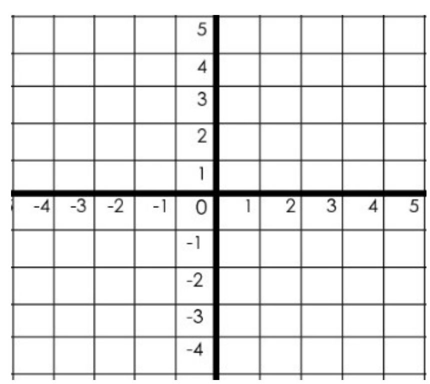
15. $y = 2x$

$\boldsymbol{x}$	$\boldsymbol{y}$
0	
1	
2	
3	



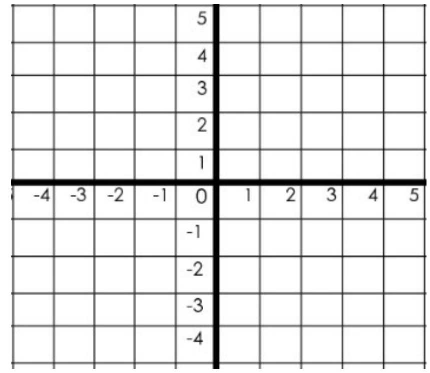
16. $y = -x$

x	y
-3	
-1	
1	
3	



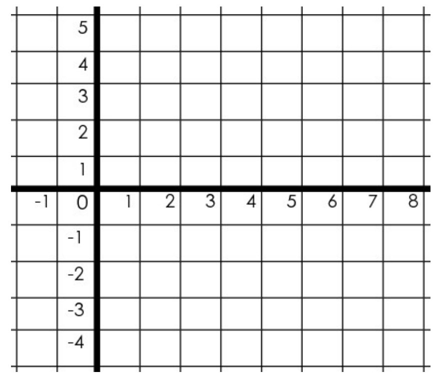
17. $y = 2x - 3$

x	y
0	
1	
2	
3	



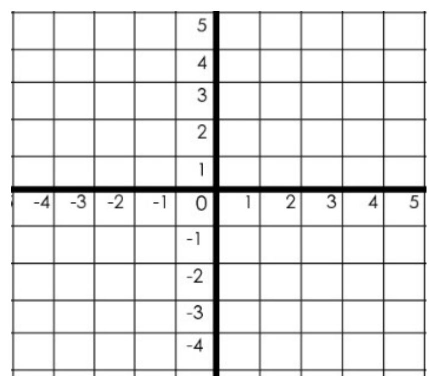
18. $y = \frac{1}{2}x + 1$

x	y
0	
2	
4	
6	



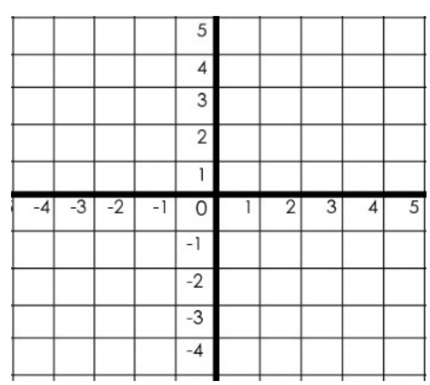
19. $y = \frac{3}{2}x - 1$

x	y
-2	
0	
2	



20. $y = -\frac{2}{3}x + 1$

x	y
-3	
0	
3	



Solving Equations

To solve an equation means to *find the value* of the variable. We solve equations by isolating the variable using opposite operations.

Opposite Operations:
Addition (+) & Subtraction (-)
Multiplication (x) and Division (÷)

Please remember…
to do the same step on
each side of the equation.

Always check your work by substitution!

Example:

Solve.

$3x - 2 = 10$

$\quad +2 \quad +2$ — Isolate 3x by adding 2 to each side.

$\frac{3x}{3} = \frac{12}{3}$ — Simplify. Isolate x by dividing each side by 3.

$x = 4$ — Simplify

Check your answer.

$3(4) - 2 = 10$ — Substitute the value in for the variable.

$12 - 2 = 10$ — Simplify

$10 = 10$ — Is the equation true? If yes, you solved it correctly!

Try these:

Solve each equation below.

1. $x + 3 = 5$
2. $w - 4 = 10$
3. $c - 5 = -8$
4. $3p = 9$
5. $-7k = 14$
6. $-x = -17$
7. $\frac{h}{3} = 5$
8. $\frac{m}{8} = 7$
9. $\frac{4}{5}d = 12$
10. $\frac{3}{8}j = 6$

11. $2x - 5 = 11$

12. $4n + 1 = 9$

13. $5j - 3 = 12$

14. $2x + 11 = 9$

15. $-3x + 4 = -8$

16. $-6x + 3 = -9$

17. $\frac{f}{3} + 10 = 15$

18. $\frac{a}{7} - 4 = 2$

19. $\frac{b+4}{2} = 5$

20. $\frac{x-6}{5} = -3$

Use substitution to determine whether the solution is correct.

21. $4x - 5 = 7$ $\quad x = 3$

23. $6 - x = 8$ $\quad x = 2$

22. $-2x + 5 = 13$ $\quad x = 4$

24. $1 - x = 9$ $\quad x = -8$

Inequalities

An inequality is a statement containing on of the following symbols.

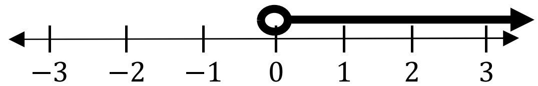
$<$ is less than　　$>$ is greater than　　$\leq$ is less than or equal to　　$\geq$ is greater than or equal to

An inequality has many solutions, and we can represent the solutions of an inequality by a set of numbers on a number line.

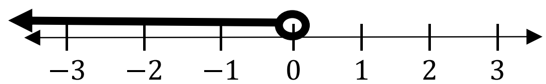
When graphing an inequality,　　$<$ and $>$ use an open circle ○　　$\leq$ and $\geq$ use a closed circle●

Examples:

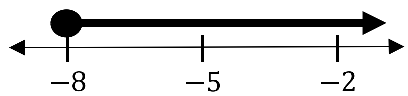
$x > 0$



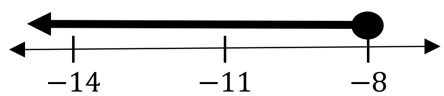
$x < 0$



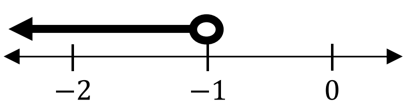
$x \geq -8$



$x \leq -8$



Practice: Write an inequality to represent the solution set that is shown in the graph.

1. 

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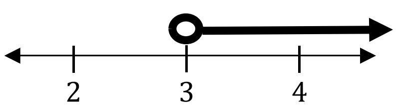
2. 

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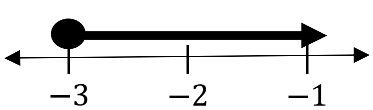
3. 

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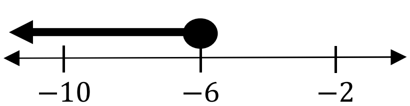
4. 

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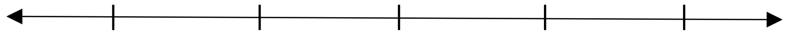
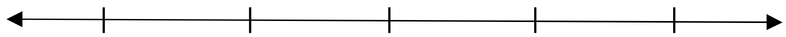
Graph each of the following inequalities on a number line.

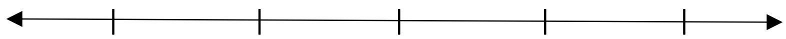
1. $x > 4$



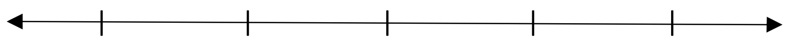
2. $k \leq -6$



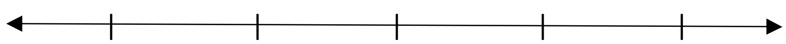
3. $5 > y$



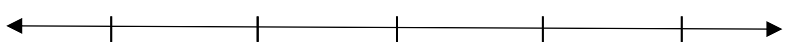
4. $j < -\frac{1}{2}$



5. $-2 \leq t$



6. $w \leq 15$



Algebraic Translations – Translating from English to Mathematics

Key words for Translations:

Add	Subtract	Multiply	Divide	Inequalities	Variable	=
Plus	Decreased	For every	Per	$<$ is less than	A number	Same as
Sum	Smaller	For each	One third	$>$ is greater than	Some number	Equals
Longer Than	Less Than	Triple	Quotient		quantity	Is
Greater Than	Difference	Multiplied	Divided by	$\leq$ is less than or equal to		Total
Together	Reduced	Of	Each part			Was
Total	Differ	Times	Half as much	$\geq$ is greater than or equal to		Result
Increased	Fewer	Twice	Split equally			Outcome
More Than	Shorter Than	Double				Answer
In all	Minus					
And	Diminished					

Examples:

A) Translate into a mathematical expression: 3 less than 5 times some number

3 **less than** 5 **times** **some number**

to subtract from	multiply	use a variable

Translation: $\mathbf{5n - 3}$

B) Translate into a mathematical statement: 3 less than 5 times some number is 22

3 **less than** 5 **times** **some number** **is** 22

to subtract from	multiply	use a variable	=

Translation: $\mathbf{5n - 3 = 22}$

C) Translate into a mathematical statement: the quotient of a number and -4, less 8, is -42

The quotient of a **number** and -4, **less** 8, **is** -42

divide a variable and a number	subtract	=

Translation: $\mathbf{\frac{n}{-4} - 8 = -42}$

D) Translate into a mathematical statement: four plus three times a number is less than or equal to 18

Four **plus** three **times** **a number** **is less than or equal to** 18

add	multiply	use a variable	$\leq$

Translation: $\mathbf{4 + 3n \leq 18}$

Practice: Translate each phrase into a mathematical statement

1. Seven plus five times a number is greater than or equal to -9

2. Eight times a number increased by 6 is 62

3. One half a number is equal to 14

4. 6 less than 8 times some number

5. A number divided by 9

6. p decreased by 5

7. Twice a number decreased by 15 is equal to -27

8. 9 less than 7 times some number is -6

9. The sum of a number and eight is less than 2

10. Eleven increased by a number is -12

Matching – Put the letter of the algebraic expression that best matches the phrase.

_______	1. two more than a number	a. $2x$
_______	2. two less than a number	b. $x + 2$
_______	3. half a number	c. $2 - x$
_______	4. twice a number	d. $x - 2$
_______	5. two decreased by a number	e. $\frac{x}{2}$

Careful! Pay attention to subtraction. The order makes a difference. Translate to an algebraic expression, then reread to check!

Word Problems

Translate each word problem into an algebraic equation, using x for the unknown, and solve. Write a "**let** $\boldsymbol{x} =$" for each unknown; write an equation; solve the equation; substitute the value for x into the let statement(s) to answer the equation.

For example:

Kara is going to Maui on vacation. She paid $325 for her plane ticket and is spending $125 each night for the hotel. How many nights can she stay in Maui if she has $1200?

Step 1: What are you asked to find? Let variables represent what you are aske to find.

How many nights can Kara stay in Maui?

Let $x =$ the number nights Kara can stay in Maui

Step 2: Write an equation to represent the relationship in the problem.

$325 + 125x = 1200$

Step 3: Solve the equation for the unknown

$$\begin{aligned} 325 + 125x &= 1200 \\ -325 \qquad &\quad - 325 \\ 125x &= 875 \\ x &= 7 \end{aligned}$$

Kara can spend 7 nights in Maui

Word Problem Practice Set

1. A video store charges a one-time membership fee of $12.00 plus $1.50 per video rental. How many videos can Stewart rent if he spends $21?

2. Bicycle city makes custom bicycles. They charge $160 plus $80 for each day that it takes to build the bicycle. If you have $480 to spend on your new bicycle, how many days can it take Bicycle City to build the bike?

3. Darel went to the mall and spent $41. He bought several t-shirts that each cost $12 and he bought 1 pair of socks for $5. How many t-shirts did Darel buy?

4. Janet weighs 20 pounds more than Anna. If the sum of their weights is 250 pounds, how much does each girl weigh?

5. Three-fourths of the student body attended the pep rally. If there were 1230 students at the pep rally, how many students are there in all?

6. Two-thirds of the Algebra students tool the H S A the first time. If 60 students took the algebra H S A, how many algebra students ae there in all?

7. The current price of a school t-shirt is $10.58. Next year the cost of a t-shirt will be $15.35. How much will the t-shirt increase next year?

8. The school lunch prices are changing next year. The cost of a hot lunch will increase $0.45 from the current price. If the next year's price is $2.60, what did the hot lunch cost this year?

9. Next year the cost of gasoline will increase $1.25 from the current price. If the cost of a gallon of gasoline next year will be $4.50, what is the current price of gasoline?

10. Sarah drove 3 hours more than Michael on their trip to Texas. If the trip took 37 hours, how long did Sarah and Michael each drive?